Solutions (w/ partial explanations)

Problem 1

Water is being pumped into a tank. The amount of water in the tank is modeled by $W(t) = 12 + 3t$ where t is the time in minutes since we started pumping water in and $W(t)$ is in gallons.

(a) What is the input of this function? What is the output of this function? *Make sure to include units.*

Input: time in minutes Output: gallons of water in tank

(b) At what rate is the water going into the tank? *Make sure to include units.*

3 gal/min

(c) How long does it take for the tank to fill up to 63 gallons? *Make sure to include units.*

↳ Solve for input ↳ Output = 63 gal

$W(t) = 63$

$12 + 3t = 63$

$-12 \quad -12$

$\frac{3t}{3} = \frac{51}{3}$

$t = 17$ min

(d) Where is the *vertical* intercept of this function? What does this point mean in the context of the problem?

12 gal; the tank is 12 gal full before water is pumped in

(e) Evaluate $W(32)$. Then, interpret what this result means in the context of the problem.

↳ Input

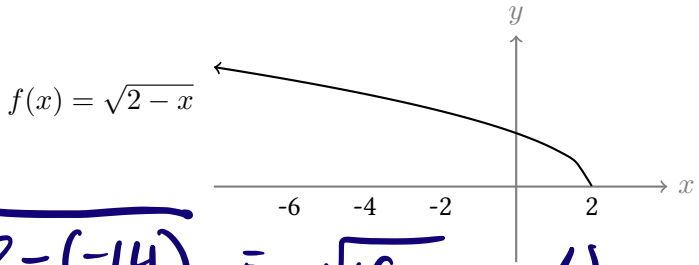
$W(32) = 12 + 3(32) = 108$

After 32 minutes the tank contains 108 gallons.

Problem 2

The function $f(x) = \sqrt{2 - x}$ is given below. What is the restricted range of $f(x)$ on the domain $[-14, -2)$?

Plug in endpts.



$f(-14) = \sqrt{2-(-14)} = \sqrt{16} = 4$

$f(-2) = \sqrt{2-(-2)} = \sqrt{4} = 2$

$[-14, 2)$ domain $\rightsquigarrow$ $(2, 4]$ range

Problem 3

Darius wants to start selling his collection to make some money. It will cost \$35 to open up an online store. He knows he can sell each item he has for \$12, but it costs him \$5 to process and ship one item each to the customers.

(a) Let $R(x)$ be the revenue Darius makes for selling x items. Let $C(x)$ be the cost function of his new business. Write down the functions:

$R(x) = 12x$ $\quad$ $C(x) = 5x + 35$

(b) Write down Darius's profit function $P(x)$ where x is how many items he sold.

$P(x) = 12x - (5x+35) = 12x - 5x - 35 = 7x - 35$

(c) If Darius wants to make at least \$200, how many items must he sell?

$P(x) = 200$

$7x - 35 = 200$

$+35 \quad +35$

$7x = 235$

$x = 33.57\ldots$

He must sell at least 34 items

(d) Solve $P(x) = 0$. Then, interpret what this result means in the context of the problem.

$P(x) = 0$

$7x - 35 = 0$

$7x = 35$

$x = 5$

He must sell 5 items to break even (make 0 profit).

Problem 4

(a) What is the equation of the line passing through $(-3, -4)$ and $(5, 2)$? You may use either slope-intercept form or point-slope form.

Slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-4)}{5-(-3)} = \frac{6}{8} = \frac{3}{4}$

pt slope form: $y + 4 = \frac{3}{4}(x+3)$ or $y - 2 = \frac{3}{4}(x-5)$

(b) What is the equation of the line parallel to the line from the previous part and that passes through $(0, 1)$?

parallel = same slope $\quad$ y-int. so

$y = \frac{3}{4}x + 1$

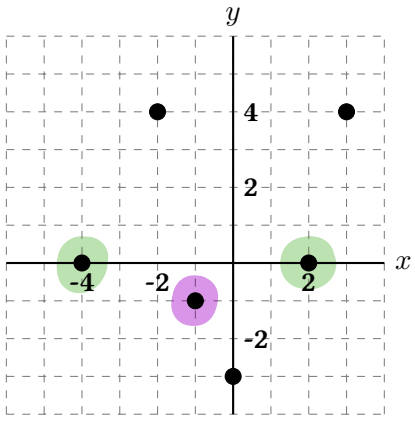
(c) What is the slope of the line perpendicular to the one from part (a)? We are only asking for the slope.

$-4/3$

(opposite reciprocal)

Problem 5

Consider the graph of a relation below.



(a) Is this relation a function or not? Briefly justify.

Yes; it passes vertical line test

(b) What is the domain of this relation?

$\{-4, -2, -1, 0, 2, 3\}$ i.e. all x vals

(c) What is the range of this relation?

$\{-3, -1, 0, 4\}$ i.e. all y-vals

(d) Evaluate this relation at $x = -1$.

-1 (see • point)

(e) Where does this relation have an output of 0?

-4 and 2 (see • points)

Problem 6

(a) What is the equation of the line that intersects the function $g(x) = x^2 - 1$ at $x = -3$ and $x = 1$?

At $x = -3$, $g(-3) = (-3)^2 - 1 = 9 - 1 = 8 \Rightarrow (-3, 8)$

At $x = 1$, $g(1) = 1^2 - 1 = 0 \Rightarrow (1, 0)$

$\text{Slope} = \frac{0-8}{1-(-3)} = -8/4 = -2$

pt slope form $\rightsquigarrow$ $y - 0 = -2(x-1)$ or $y - 8 = -2(x+3)$

(b) What is the average rate of change of the function $h(x) = 3 + x^2$ on the interval $[0, 3]$?

$\text{AROC} = \text{Slope} = \frac{h(3)-h(0)}{3-0}$

$\frac{h(3)-h(0)}{3-0} = \frac{(3+3^2)-(3+0^2)}{3} = \frac{12-3}{3} = \frac{9}{3} = \boxed{3}$

Problem 7

Consider the following table below to answer (a)-(c)

x	-2	0	2	4
$h(x)$	1.25	2	5	9

+2 +2 +2

+.75 +3 +4

(a) Is the following table a linear function? If so, find the equation for it.

No: the rate of change b/w the first two points is $\frac{.75}{2}$, between the second two is $\frac{3}{2}$, b/w the last two is $4/2$. All different!

(b) What is the average rate of change of $h(x)$ in the interval $[-2, 4]$?

$$\frac{h(4) - h(-2)}{4 - (-2)} = \frac{9 - 1.25}{4 + 2} = \frac{7.725}{6} = 1.29\ldots$$

Problem 8

Graph the equation of that line that is **perpendicular** to the line $3x + 2y = 8$ and has y-intercept at $(0, -2)$.

You may need to rescale the grid marks. Make sure to specify the number on your grids.

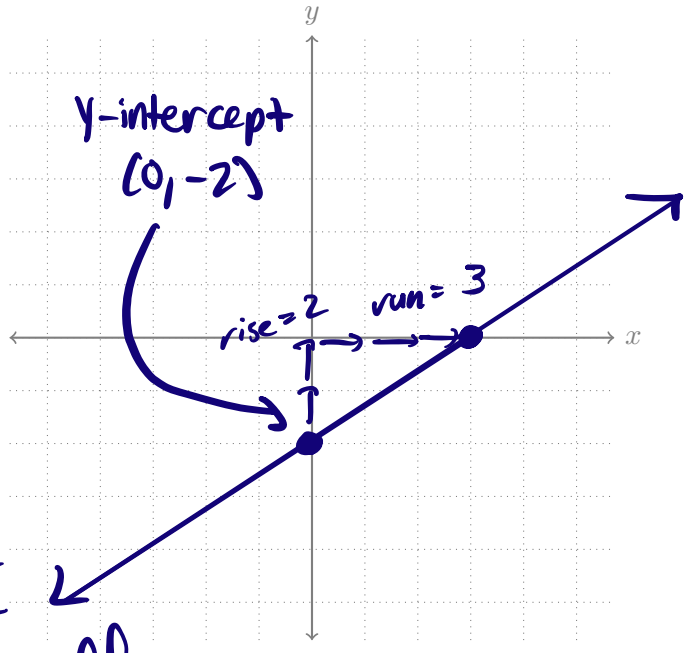
$3x + 2y = 8$

$-3x \quad -3x$

$\frac{2y}{2} = \frac{-3x + 8}{2}$

$y = \frac{-3}{2}x + 4$

↳ Slope. But we want perpendicular so our new line should have slope = $2/3$.



Problem 9

(a) What is the domain and the range of $f(x) = 3x - 2$?

$D = (-\infty, \infty)$

$R = (-\infty, \infty)$

(b) What is the domain and the range of $g(x) = \sqrt{x-3}$?

$D = [3, \infty)$

$R = [0, \infty)$

(c) What is the domain and the range of $h(x) = \frac{1}{x^2-16}$?

$= \frac{1}{(x+4)(x-4)}$

or: if $x^2 - 16 = 0$ then $x = \pm 4$

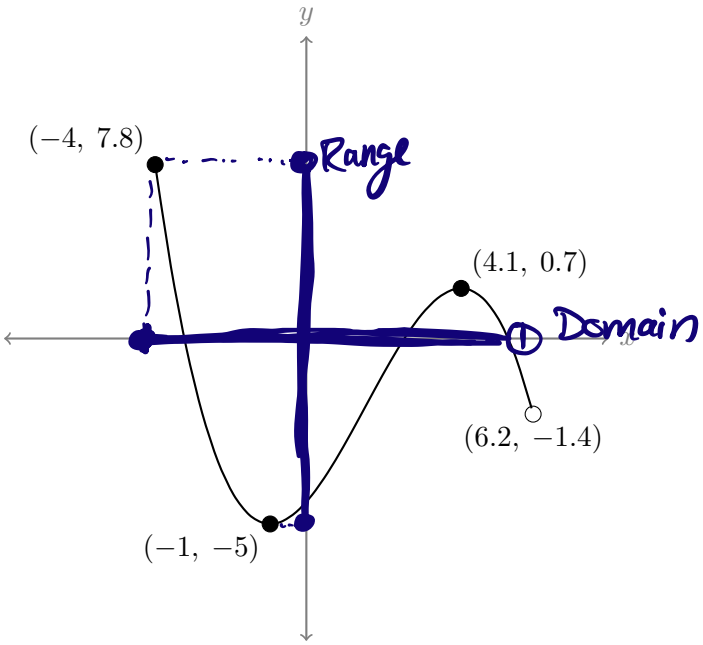
$D = (-\infty, -4) \cup (-4, 4) \cup (4, \infty)$

i.e. all numbers except -4 and 4

$R = (-\infty, 0) \cup (0, \infty)$

i.e. all numbers except 0

(d) What is the domain and the range of the following function graphed below?



$D = [-4, 6.2)$ $R = [-5, 7.8]$